EXERCISE I

$X_1, \ldots, X_n$: I.I.D.

$$\forall n, \quad F_{X_n} = \left\{ 1 - \left(1 - \frac{1}{n} \right)^{nx} \right\} \mathbf{1}(\mathbf{x} > \mathbf{0})$$

Prove the following:

$$X_n \xrightarrow[n \to \infty]{d} X, \quad X \sim \xi(1) \quad (: 1 - \exp(-x) \quad (x \geq 0))$$

EXERCISE II

$$\forall n, \quad X_n \overset{iid}{\sim} Bi\left(n, \ \frac{\lambda}{n} \right)$$

where $\lambda > 0$. Prove the following:

$$X_n \xrightarrow[n \to \infty]{d} Po(\lambda)$$

EXERCISE III

Now we have:

$$X_n = X + y_n, \quad E[y_n] = \frac{1}{n}, \quad Var(y_n) = \frac{\sigma^2}{n}.$$

Prove the following:

$$X_n \xrightarrow[n \to \infty]{p} X$$