TA Session for Econometrics I 2025

8.2

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June 11, 2025

1. OLS vs Maximum Likelihood Estimation

2. Parametric Estimation Methods

OLS vs Maximum Likelihood Estimation

Consider the linear model:

$$y = X\beta + u, \quad u \sim \mathcal{N}(0, \sigma^2 I)$$

Ordinary Least Squares (OLS):

- Minimises the sum of squared residuals:

$$\hat{\beta}_{\mathsf{OLS}} = \arg\min_{\beta} (y - X\beta)^\top (y - X\beta)$$

- No distributional assumption needed for consistency.
- Simple, intuitive, and unbiased under Gauss-Markov conditions.

Maximum Likelihood Estimation (MLE):

- Assumes $u \sim \mathcal{N}(0, \sigma^2 I)$.
- Maximises the likelihood function:

$$\mathcal{L}(\beta, \sigma^2) = (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2}(y - X\beta)^\top (y - X\beta)\right)$$

- Yields the same $\hat{\beta}$ as OLS under normality, but also estimates $\hat{\sigma}^2$.

Parametric Estimation Methods

Parametric methods rely on specific assumptions about the distribution of the data.

- Assume a known functional form for the population (e.g., Normal distribution).
- A finite number of parameters to estimate (e.g., β, σ^2 in linear regression).
- Examples:
  - Maximum Likelihood Estimation (MLE)
  - Bayesian methods with known likelihoods

Pros: Efficient and interpretable when assumptions are correct.
Cons: Sensitive to misspecification of the distribution.

Unlike OLS, which does not require a distributional assumption for consistency, MLE is a parametric method and fully depends on the assumption of normality.

Parametric approaches are powerful but can lead to biased or inconsistent results if the assumed model is incorrect.

Non-parametric $\neq$ Assumption-free

Misconception: Non-parametric methods are completely assumption-free.

Reality:

- Non-parametric methods do not assume a specific form (e.g., Normal), but still rely on key assumptions:
  - Choice of **kernel function** (e.g., Gaussian, Epanechnikov)
  - **Bandwidth selection**, which controls smoothness
  - Often assume continuity or smoothness of the true distribution
- Therefore, they are flexible but not entirely assumption-free.

"Non-parametric methods are named as such because they do not assume a fixed number of parameters or a specific parametric form of the underlying distribution."

"It does not mean they are free of assumptions — rather, they avoid assuming a particular distributional shape like the normal or Poisson."